Discrete Mathematics 46 (1983) 205–206
North-Holland

NOTE

A CORRESPONDENCE BETWEEN ORDERED TREES AND NONCROSSING PARTITIONS

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Received 14 January 1983

The *Narayana* numbers

$$\frac{1}{n}\binom{n}{k}\binom{n}{k-1}$$

appear twice in Volume 31 of Discrete Mathematics: They count the ordered trees with n edges (i.e. $n+1$ nodes) and k leaves [1] and the noncrossing partitions of $\{1,\ldots,n\}$ into k blocks [2]. (In such a partition the existence of four numbers $a<b<c<d$ such that a and c are in one block and b and d are in another block is forbidden.) The aim of this note is to exhibit a bijection between these combinatorial objects.

1. Given an ordered tree with n edges, we attach the numbers $1,\ldots,n$ to the edges in the following way: We traverse the tree in preorder (visit the root, then traverse its subtrees from left to right) and label an edge whenever we see it first with the smallest number not yet used. From this labelling we construct a noncrossing partition as follows: The chain, starting from the root and containing the edge labelled n forms one set. After deleting that chain, we have an ordered forest and follow the same principle: We take the longest chain containing the highest number that is yet available; this gives the second set, and so on.

2. Now we produce the inverse map: Given a noncrossing partition, we draw a (monotonically labelled) chain using that set containing the number n. Now we take that block containing the highest number not yet used, form a chain and attach it to the first chain 'where it fits': If this new chain starts with number a, it fits at the node of the first chain which is between labels b, c such that $b<a<c$. If such a situation does not exist, we attach the second chain at the root of the first

0012-365X/83/$3.00

chain. This process will be repeated: The next chain has to lie 'left' to the already constructed tree; there is a unique node where it fits.

It is indeed a matter of routine to show that these constructions work.

References

[1] N. Dershowitz and S. Zaks, Enumerations of ordered trees, Discrete Math. 31 (1980) 9–28.
[2] P.H. Edelman, Chain enumeration and non-crossing partitions, Discrete Math. 31 (1980) 171–180.